Int. J. Heat Mass Transfer. Vol. 3, pp. 77–83. Pergamon Press 1961. Printed in Great Britain.

TURBULENT HEAT TRANSFER IN LIQUID METALS—FULLY DEVELOPED PIPE FLOW WITH CONSTANT WALL TEMPERATURE

N. Z. AZER* and B. T. CHAO†

*University of Alexandria, UAR (Egypt), and †University of Illinois, Urbana, Illinois

(*Received* 11 *July* 1960, *and in revised form* 27 *January* 1961)

Abstract—This paper is a continuation of the authors' earlier work on a mechanism of turbulent heat transfer in liquid metals [1]. Nusselt number and temperature profile for low Prandtl number fluids of constant properties flowing in a smooth pipe with constant wall temperature have been evaluated. Use is made of the theoretical expression for the ratio of eddy diffusivities for heat and momentum deduced in [1]. For practical calculation of film coefficient of heat transfer, an interpolation formula is proposed:

$$N_{Nu} = 5 + 0{\cdot}05\, N_{Pr}^{0\cdot25}\, N_{Pe}^{0\cdot77}$$

which fits the calculated data with a maximum deviation of less than 11 per cent for $N_{Pr} < 0{\cdot}1$ and $N_{Pe} < 15\,000$. Temperature profiles for several Prandtl and Reynolds numbers were compared with the case of constant wall flux.

Résumé—Cet article est la suite d'un travail précédent des auteurs sur le mécanisme de la transmission de chaleur turbulente dans les métaux liquides. Le nombre de Nusselt et le profil des températures sont déterminés pour des fluides, à propriétés constantes et faible nombre de Prandtl, s'écoulant dans une conduite lisse à température de paroi constante. On a utilisé l'expression théorique obtenue en [1] pour la détermination du rapport des diffusivités turbulentes de la chaleur et de la masse. Pour le calcul pratique du coefficient de transmission pariétale de chaleur, une formule d'interpolation est proposée

$$N_{Nu} = 5 + 0{,}05\, N_{Pr}^{0{,}25}\, N_{Pe}^{0{,}77}$$

dont les résultats diffèrent des données calculées de moins de 11% pour $N_{Pr} < 0{,}1$ et $N_{Pe} < 15\,000$. Les profils de température ont été comparés, pour plusieurs nombres de Prandtl et de Reynolds, avec le cas d'un flux de paroi constant.

Zusammenfassung—Eine frühere Arbeit der Autoren über den turbulenten Wärmeübergang in flüssigen Metallen wird fortgeführt. Für Flüssigkeiten konstanter Eigenschaften und kleiner Prandtlzahlen, die in glatten Rohren von gleichbleibender Wandtemperatur fliessen, wurden die Nusseltzahlen und die Temperaturprofile ermittelt. Von dem in [1] abgeleiteten theoretischen Ausdruck für das Verhältnis der Austauschgrössen von Wärme und Impuls wird Gebrauch gemacht. Die praktische Berechnung der auftretenden Wärmeübergangszahlen kann nach der Interpolationsformel

$$Nu = 5 + 0{,}05\, Pr^{0{,}25}\, Pe^{0{,}77}$$

erfolgen. Im Bereich $Pr < 0{,}1$ und $Pe < 15\,000$ ist die Maximalabweichung von den errechneten Werten geringer als 11%. Für verschiedene Prandtl- und Reynoldszahlen wurden die Temperaturprofile mit denen bei konstantem Wärmefluss an der Wand verglichen.

Аннотация—Настоящая статья является продолжением ранней работы автора по исследованию механиэма турбулентного переноса тепла в жидких металлах [1]. Вычислены критерий Нуссельта и профиль температуры для потоков с низким числом Прандтля и постоянными свойствами в гладкой трубе, температура стенок которой постоянна. Использовано теоретическое выражение, приведенное в [1] для отношения коэффициентов вихревой диффуэии тепла и количества движения. Предложена интерполяционная формула для практического вычисления коэффициента плёночного теплообмена:

$$N_{Nu} = 5 + 0{,}05\, N_{Pr}^{0{,}25}\, N_{Pe}^{0{,}77}$$

которая согласуется с вычисленными данными с максимальным отклонением в 11% для числа $N_{Pr} < 0,1$ и $N_{Pe} < 15\,000$. Дано сравнение профилей температуры для нескольких чисел Прандтля и Рейнольдса со случаем постоянного потока от стенки.

NOMENCLATURE

D, pipe inside diameter, $2r_w$ (ft);
h, surface conductance (Btu/ft² h degF)
k, thermal conductivity of fluid (Btu/ft h degF);
q_w, heat flux at pipe wall (Btu/ft² h);
r_w, inside radius of pipe (ft);
u, axial velocity (ft/h);
ϵ_H, eddy diffusivity for heat transfer (ft²/h);
ϵ_M, eddy diffusivity for momentum transfer (ft²/h);
κ, thermal diffusivity of fluid (ft²/h);
γ, kinematic viscosity of fluid (ft²/h).

Dimensionless quantities

N_{Nu}, Nusselt number, $\frac{hD}{k}$;
N_{Pe}, Peclet number, $N_{Re}\,N_{Pr} = \frac{u_b D}{\kappa}$;
N_{Pr}, Prandtl number, $\frac{\nu}{\kappa}$;
N_{Re}, Reynolds number, $\frac{u_b D}{\nu}$;
$U = \frac{u}{u_b}$;
$Z = \frac{r}{r_w}$;
$\alpha = \frac{\epsilon_H}{\epsilon_M}$, ratio of eddy diffusivity of heat to momentum.

Subscripts

b, bulk;
c, center of pipe;
w, wall.

1. INTRODUCTION

The effect of wall thermal conditions on surface conductance for fully developed turbulent flow in pipes was first reported by Reichardt [2]. For fluids whose Prandtl number is comparable to, or greater than, that of air, Reichardt found only small differences in heat transfer coefficient when results for constant wall flux were compared with those for constant wall temperature. Reichardt's findings have recently been confirmed by Siegel and Sparrow [3]. For fluids of low Prandtl number, such as liquid metals, Seban and Shimazaki [4] reported that the influence of wall-temperature variation was significant. In the latter analysis, the eddy diffusivity for heat ϵ_H was assumed to be identical to the eddy diffusivity for momentum, ϵ_M. However, experimental data indicate that this assumption is, in general, not valid [5, 6]. Based on a modification of Prandtl's mixing-length hypothesis, the authors [1] obtained an expression for the diffusivity ratio $\alpha\ [= (\epsilon_H/\epsilon_M)]$. It depends on N_{Re}, N_{Pr} as well as radial location across the pipe. The theoretical expression gives fair agreement with available experimental data. In the following, Seban and Shimazaki's analysis is repeated but without the assumption of α being unity.

2. NUSSELT NUMBER AND TEMPERATURE PROFILE

For fully developed velocity and temperature distribution in constant property fluids flowing in pipes, with dissipation effects and axial conduction neglected, Seban and Shimazaki [4] showed that the Nusselt number and temperature profile are given respectively by:

$$N_{Nu} = \frac{N_{Pr}}{\psi(1)} \tag{1}$$

and

$$\frac{t_w - t}{t_w - t_c} = 1 - \frac{\psi(Z)}{\psi(1)} \tag{2}$$

where

$$\psi(Z) = N_{Pr} \int_0^Z \frac{\phi(Z)\,\mathrm{d}Z}{Z[1 + \alpha(\epsilon_M/\nu)\,N_{Pr}]} \tag{3}$$

and

$$\phi(Z) = \int_0^Z U\left(\frac{t_w - t}{t_w - t_c}\right) Z\,\mathrm{d}Z. \tag{4}$$

Since the evaluation of $\phi(Z)$ requires knowledge of the very temperature profile being sought, Seban and Shimazaki employed an iterative process in which the temperature profile obtained by Martinelli [7] for the case of constant wall flux was used as the first approximation. The ratio (ϵ_M/ν) was calculated from the universal velocity profile suggested by von Kármán [8]. They expressed their calculated values of Nusselt number by an interpolation formula of the form:

$$N_{Nu} = 5\cdot0 + 0\cdot025\, N_{Pe}{}^{0\cdot8}. \qquad (5)$$

In the present analysis, values of α are calculated from:

$$\alpha = \frac{\epsilon_H}{\epsilon_M} = \frac{1 + 135\, N_{Re}^{-0\cdot45} \exp\,[-(y/r_w)^{0\cdot25}]}{1 + 380\, N_{Pe}^{-0\cdot58} \exp\,[-(y/r_w)^{0\cdot25}]} \qquad (6)$$

which is formulated from Prandtl's mixing-length hypothesis but modified for a continuous exchange of momentum and energy during the flight of the eddy. A detailed exposition is given in Ref. 1. Also, for reasons there expounded, the ratio (ϵ_M/ν) was evaluated directly from the velocity profile measurements of Nikuradse. In the first iteration, use was made of the temperature profile previously calculated for constant wall flux, also given in Ref. 1. Sufficient convergence was obtained after two iterations. Table 1 gives the results of computation. It is seen that the difference between the first and second iteration is small. Corresponding values for the constant wall flux case were also listed. Invariably, the Nusselt number is smaller under the constant wall temperature condition. Percentagewise, the difference is larger for smaller Prandtl numbers. For the highest Prandtl number considered, i.e. $N_{Pr} = 0\cdot100$, the difference becomes insignificant at large Reynolds numbers. These findings are in accord with those reported by Seban and Shimazaki. Fig. 1 compares the results obtained in this analysis with those reported in Ref. 4. The lower values of Nusselt number predicted are primarily due to the abandonment of the assumption that eddy diffusivities for heat and momentum are identical. While the use of such assumption definitely results in higher Nusselt number for both the constant flux and constant temperature wall condition, their ratio is not appreciably affected by variations in α, particularly for low N_{Pr} fluids. (See last two columns in Table 1.)

As is also seen from the Table, the Nusselt number tends to be insensitive to changes in N_{Re} for vanishingly small Prandtl numbers. It ranges only from 5·16 to 5·35 while N_{Re} varies

Table 1. *Calculated values of Nusselt number*

N_{Re}	N_{Pr}	N_{Pe}	$[N_{Nu,b}]_{q_w = \text{const.}}$	$[N_{Nu,b}]_{t_w = \text{const.}}$ 1st Iteration	$[N_{Nu,b}]_{t_w = \text{const.}}$ 2nd Iteration	$[N_{Nu,b}]_{t_w = \text{const.}} / [N_{Nu,b}]_{q_w = \text{const.}}$ Present analysis	Seban and Shimazaki* ($\alpha = 1$)
$4\cdot34 \cdot 10^4$	0·000	0	7	4·98	5·16	0·74	—
	0·001	43·4	7·04	5·01	5·18	0·74	0·73
	0·010	434	8·01	5·82	6·01	0·75	0·77
	0·100	4340	26·07	22·14	22·54	0·85	0·96
$3\cdot96 \cdot 10^5$	0·000	0	7	5·08	5·25	0·75	—
	0·001	396	7·46	5·48	5·66	0·76	0·76
	0·010	3960	16·90	14·12	14·37	0·85	0·83
	0·100	39600	107·99	102·48	102·41	0·95	0·89
$3\cdot24 \cdot 10^6$	0·000	0	7	5·16	5·34	0·76	—
	0·001	3240	12·51	10·01	10·30	0·82	—
	0·010	32400	65·17	60·49	63·55	0·98	—

* Taken from Fig. 2 in Ref. 4.

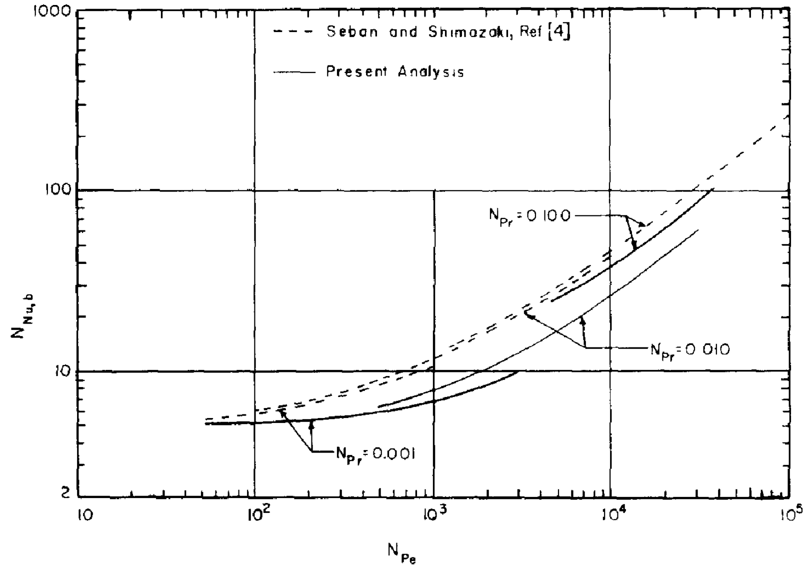

FIG. 1. Calculated values of Nusselt number at constant wall temperature.

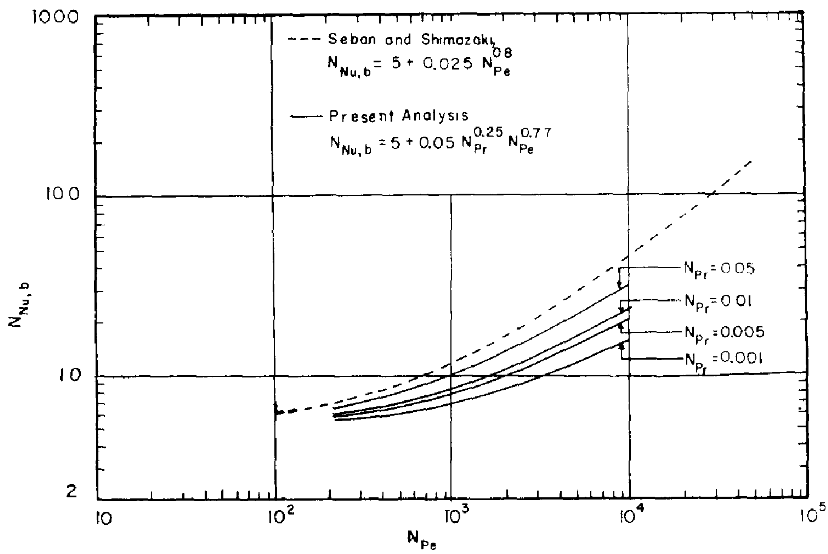

FIG. 2. Comparison of Seban and Shimazaki's equation with the present analysis.

nearly a hundredfold. It is interesting to note that the corresponding value of Nusselt number is 7 under constant wall flux condition. If, in addition, as $N_{Re} \to \infty$, the limiting Nusselt number has been found to be 5·88 which is the result obtained after three iterations. Under conditions of constant wall flux,

$$\lim N_{Nu,\,b} = 8$$
$$N_{Pr} \to 0$$
$$N_{Re} \to \infty.$$

Equation (5) asserts that the Nusselt number depends solely on Peclet number. The present analysis, however, reveals an independent Prandtl number effect. For $N_{Pr} < 0{\cdot}1$ and $N_{Pe} < 15\,000$ which cover the usual range of turbulent liquid metal heat transfer encountered in practice, the computed data shown in Table 1 could be represented by:

$$N_{Nu,\,b} = 5 + 0{\cdot}05\, N_{Pr}^{0{\cdot}25}\, N_{Pe}^{0{\cdot}77} \qquad (7)$$

which gives a maximum deviation of less than 11 per cent. Fig. 2 compares graphically Seban and Shimazaki's equation (5) with the proposed relation (7).

The difference in temperature profile for constant heat flux and constant wall temperature is illustrated in Fig. 3(a, b and c). For a given wall to center line temperature difference, the constant

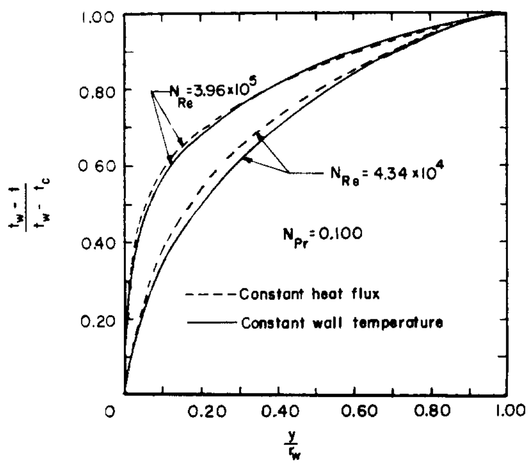

FIG. 3(a) Temperature profiles under conditions of constant wall flux and constant wall temperature. $N_{Pr} = 0{\cdot}1$.

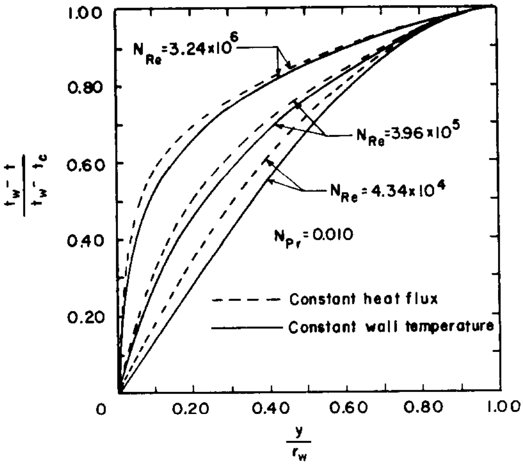

FIG. 3(b) Temperature profiles under conditions of constant wall flux and constant wall temperature. $N_{Pr} = 0{\cdot}01$.

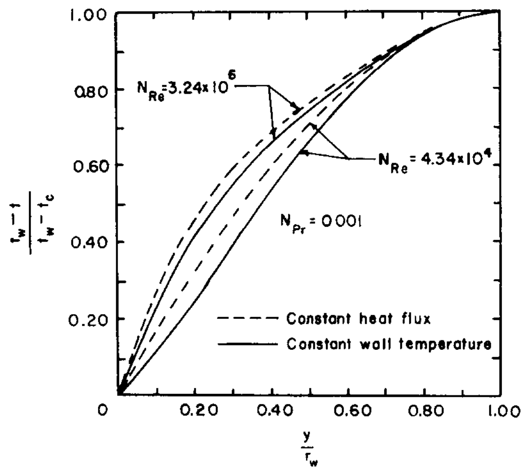

FIG. 3(c) Temperature profiles under conditions of constant wall flux and constant wall temperature. $N_{Pr} = 0{\cdot}001$.

heat flux condition yields a steeper temperature gradient at wall and thus a higher rate of heat transfer. The deviation is most pronounced at the smallest N_{Pr}. For a given N_{Pr}, the deviation becomes less at higher N_{Re}.

Experimental data of heat transfer coefficient for fully developed turbulent flow of liquid metals in circular tubes under constant wall temperature conditions are practically non-existent. Gilliland *et al.* [9] reported some results for mercury. Heating was done by dropwise condensation of steam on the outside of a vertical nickel tube, 0·319 in i.d. and 14·4 in long, thus approximating constant wall temperature condition. The Nusselt number reported was not for fully developed conditions but rather an overall average. For purpose of comparison, a small correction was introduced to take into account the finite length to diameter ratio of the test section [$(L/D) = 45$ in Gilliland's experiments] using the theoretical results of Deissler [10]. Fig. 4 compares their data so modified with the proposed correlation. Should

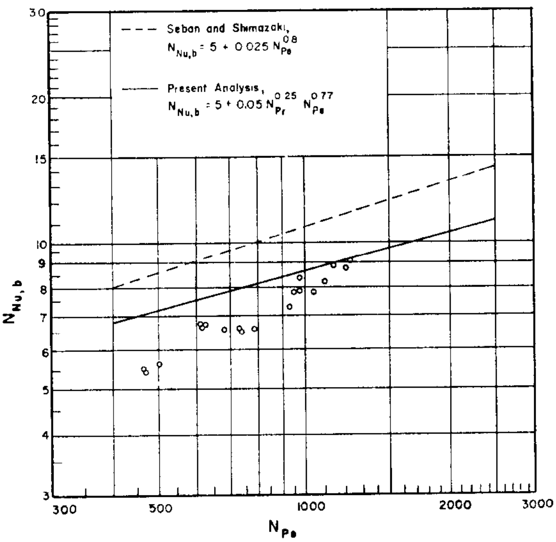

FIG. 4. Comparison of re-evaluated experimental data due to Gilliland *et al.* with the present analysis.

their raw data be used, the agreement would be even better. Shown also is a plot of equation (5) due to Seban and Shimazaki. At the present time, the authors do not construe that the proposed equation (7) gives a better correlation than equation (5). More experimental data are needed before a definite conclusion could be made.

Heating and cooling coefficients for mercury with and without sodium addition have been measured by Doody and Younger [11]. Their arrangement resulted in a wall condition somewhere between uniform wall temperature and uniform wall flux for some of the test runs. Others approximated more closely to uniform flux condition. Their heat balance indicated discrepancies as great as 140 per cent. Bailey *et al.* [12] reported cooling coefficients for mercury flowing in 0·437 in i.d. steel tube. At low Peclet numbers, their method of cooling resulted in approximately constant wall temperature condition. Unfortunately, the reliability of their data was open to question since no provision was made for mixing before exit temperatures were measured. Similar difficulty occurred in Doody and Younger's experiments. For these reasons, no attempt was made to compare these data with the theoretical predictions of equation (7).

3. FLUID BULK TEMPERATURE

It would be interesting to compare the fluid bulk temperatures under conditions of constant wall flux and constant wall temperature. A general expression for the temperature-difference ratio is:

$$\vartheta = \frac{t_w - t_b}{t_w - t_c} = \frac{\int_0^1 \left(\frac{t_w - t}{t_w - t_c}\right) UZ\,\mathrm{d}Z}{\int_0^1 UZ\,\mathrm{d}Z} \tag{8}$$

which holds true irrespective of wall conditions. The numerator of equation (8) is seen to be $\phi(1)$. In Ref. 1, it has been demonstrated that the value of the integral $\int_0^Z UZ\,\mathrm{d}Z$ is insensitive to changes in N_{Re} and could be closely approximated by $(Z^{1\cdot 75}/2)$ for N_{Re} ranging from 4×10^3 to over 3×10^6. Using this approximation, the denominator of equation (8) assumes a constant value of 1/2. The results of computation are briefly indicated below. Details may be found in [13].

The temperature-difference ratio, ϑ, defined by equation (8) has been found to be invariably smaller for the constant wall temperature condition when compared to the constant flux case. For instance, at $N_{Re} = 3{\cdot}96 \times 10^5$, $N_{Pr} = 0{\cdot}001$, ϑ is 0·470 and 0·504 respectively for the two wall conditions. However, as N_{Pr} increases, the ratio tends to be equal. As an illustration, we may cite that for $N_{Pr} = 0{\cdot}1$ the corresponding ratios are 0·740 and 0·742 at the said N_{Re}.

Under the condition of constant wall flux, Martinelli [7] computed and reported values of

ϑ for $\alpha = 1$. Values of ϑ have also been evaluated by Lykoudis and Touloukian [14] using variable α. However, unlike the authors' expression for diffusivity ratio, Lykoudis and Touloukian's expression for α shows only Prandtl number dependency. The following compares the temperature-difference ratios evaluated for $N_{Re} = 3{\cdot}96 \times 10^5$.

$$\left[\vartheta = \frac{t_w - t_b}{t_w - t_c}\right]_{q_w = \text{const.}}$$

	Martinelli	Lykoudis and Touloukian	Present analysis
$N_{Pr} = 0{\cdot}1$	0·79	0·76	0·742
$N_{Pr} = 0{\cdot}001$	0·60	0·55	0·504

For very small N_{Pr}, ϑ tends to become insensitive to Reynolds number variation. In the limit, as $N_{Pr} \to 0$, $N_{Re} \to \infty$, $[\vartheta]_{q_w = \text{const.}} = 0{\cdot}5$ and $[\vartheta]_{t_w = \text{const.}} = 0{\cdot}446$. The former has also been reported in Ref. 14.

REFERENCES

1. N. Z. Azer and B. T. Chao, A mechanism of turbulent heat transfer in liquid metals. *Int. J. Heat Mass Transfer*, **1**, 121 (1960).
2. H. Reichardt, Heat transfer through turbulent friction layers, *N.A.C.A. TM* 1047 (1943).
3. R. Siegel and E. M. Sparrow, *J. Heat Transfer*, C**82**, 152 (1960).
4. R. A. Seban and T. T. Shimazaki, *Trans. Amer. Soc. Mech. Engrs*, **73**, 803 (1951).
5. S. E. Isakoff and T. B. Drew, *General Discussion on Heat Transfer*, p. 405, Institution of Mechanical Engineers, London (1951).
6. H. E. Brown, B. H. Amstead and B. E. Short, *Trans. Amer. Soc. Mech. Engrs*, **79**, 279 (1957).
7. R. C. Martinelli, *Trans. Amer. Soc. Mech. Engrs* **69**, 947 (1949).
8. T. von Kármán, *Trans. Amer. Soc. Mech. Engrs* **61**, 705 (1939).
9. E. R. Gilliland, R. J. Musser and W. R. Page, *General Discussion on Heat Transfer*, p. 402, Institution of Mechanical Engineers, London (1951).
10. R. G. Deissler, Analysis of turbulent heat transfer and flow in entrance regions of smooth passages, *N.A.C.A. TN* 3016 (1953).
11. T. C. Doody and A. H. Younger, *Heat Transfer Coefficients for Liquid Mercury and Dilute Solutions of Sodium in Mercury in Forced Convection* p. 77, Preprints of papers presented at the American Institute of Chemical Engineers Meeting, Atlantic City, New Jersey, (1951).
12. D. L. R. Bailey, W. F. Cope, and G. C. Watson, Heat transfer to mercury, *Heat Division Paper* No. 13, *Mech. Engng Res. Lab., E. Kilbride, Glasgow* (1952).
13. N. Z. Azer, Ph.D. Thesis, University of Illinois (1959).
14. P. S. Lykoudis and Y. S. Touloukian, *Trans. Amer. Soc. Mech. Engrs*, **80**, 653 (1958).